

# Foundations of Math

## CHAPTER 1 - PART 1

NAME:

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# Properties of Real Numbers

	Property	Example
1.	<b>Commutative Property of Addition</b>	
2.	<b>Commutative Property of Multiplication</b>	
3.	<b>Associative Property of Addition</b>	
4.	<b>Associative Property of Multiplication</b>	
5.	<b>Distributive Property</b>	
6.	<b>Additive Identity Property</b>	
7.	<b>Multiplicative Identity Property</b>	
8.	<b>Additive Inverse Property</b>	
9.	<b>Multiplicative Inverse Property</b>	
10.	<b>Zero Property</b>	
11.	<b>Closure Property</b>	

Identify the property used in the following examples:

1)  $t + 0 = t$

\_\_\_\_\_

2)  $1m = m$

\_\_\_\_\_

3)  $(-3 + 4) + 5 = -3 + (4 + 5)$

\_\_\_\_\_

4)  $np = pn$

\_\_\_\_\_

5)  $\frac{4}{3} * \frac{3}{4} = 1$

\_\_\_\_\_

6)  $(3 * 8)4 = 3(8 * 4)$

\_\_\_\_\_

7)  $p + q = q + p$

\_\_\_\_\_

8)  $2 + 6 = 8$

\_\_\_\_\_

9)  $a * 0 = 0$

\_\_\_\_\_

10)  $7 + -7 = 0$

\_\_\_\_\_

11)  $2(y - 7) = 2 * y - 2 * 7$

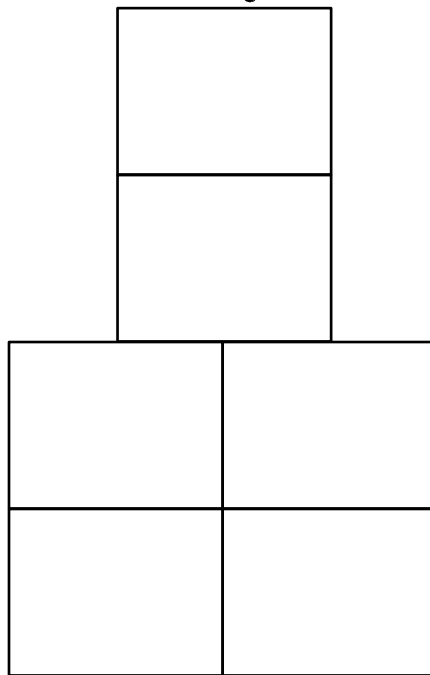
\_\_\_\_\_

**REMEMBER :**

Commutative refers to \_\_\_\_\_

Associative refers to \_\_\_\_\_

# Order of Operations



Let's try it!

1)  $(4^2 - 2 + 2) - 8 - 6$

5)  $(2 \div 1^2 + 2) + 1$

2)  $(1 - 6) + 2$

6)  $5^2 * (8 - 8) - 6$

3)  $(5 + 7 + 2)$

7)  $(7 * 2) - 6$

4)  $(2 + 8) * 7$

8)  $9 * (9 \div 3^2) + 9 + 4$



# BASIC SUBSTITUTION

What does substitution mean?

What are some examples of “substitution” in the real world?

In math, we substitute by replacing \_\_\_\_\_ with \_\_\_\_\_.

Let's try it!

Evaluate each using the values given.

1)  $y \div 2 + x$ ; for  $x = 1$  and  $y = 2$

2)  $p^2 + m$ ; for  $m = 1$  and  $p = 5$

3)  $a - 5 - b$ ; for  $a = 10$  and  $b = 4$

4)  $m + p \div 5$ ; for  $m = 1$  and  $p = 15$

5)  $(6 + h^2 - j) \div 2$ ; for  $h = 6$  and  $j = 4$

6)  $z(x + y)$ ; for  $x = 6$ ,  $y = 8$ , and  $z = 6$

7)  $p^2m \div 4$ ; for  $m = 4$  and  $p = 7$

8)  $y - (z + z^2)$ ; for  $y = 10$  and  $z = 2$



Name: \_\_\_\_\_

Notes #3

# Rational or Irrational?

Rational numbers- \_\_\_\_\_

ex)

Irrational numbers- \_\_\_\_\_


ex)

**Rational Numbers can be divided into different categories:**

Natural Numbers=

Whole Numbers=

Integers=

  
Name the set(s) of numbers to which each number belongs:

2.8	38	$\frac{-17}{31}$	0	$\sqrt{10}$	-46	$\frac{2}{3}$	$3.\bar{3}$	-0.002	$12\frac{1}{2}$	7.26841973 ...
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Natural:

Whole:

Integers:

Rational:

Irrational:

Which of the following is irrational?  $\frac{6}{11}$ ,  $\frac{2}{5}$ ,  $\sqrt{10}$

What is a counterexample?

**Is each statement true or false? If the statement is false, give a counterexample.**

**1)** Every whole number is an integer.

**3)** Every whole number is a natural number.

**2)** Every integer is a whole number.

**4)** Every natural number is a rational number.

**5)** The product of two rational numbers is rational.

**6)** The sum of two irrationals is rational.

Name: \_\_\_\_\_

Notes #4

# SOLVING ONE STEP EQUATIONS

What is so special about an equation?

Something without an equal sign is called an \_\_\_\_\_.

An equation must always remain \_\_\_\_\_.

What you do to one side of the equation, \_\_\_\_\_.

**\*\*If you subtract the same number from each side of the equation, the two sides remain EQUAL!\*\***

**\*\*If you add the same number from each side of the equation, the two sides remain EQUAL!\*\***

How do you undo addition? (In other words, what is the opposite of addition) \_\_\_\_\_

How do you undo subtraction? (In other words, what is the opposite of subtraction) \_\_\_\_\_

## EXAMPLE 1

Solve:  $x + 5 = 11$ . Solve & identify the property used.



This is asking us  
"What number plus 5  
equals 11?"

## EXAMPLE 2

Solve  $b - 28 = 22$ . Solve & identify the property used.

## EXAMPLE 3

Solve  $8.6 = n + 7.1$ . Solve & identify the property.

## EXAMPLE 4

Solve  $15 = -12 + t$ . Solve & identify the property.

WHEN WE SOLVE, WE ARE ISOLATING THE VARIABLE  
(IN OTHER WORDS, WE ARE TRYING TO GET THE LETTER BY ITSELF)

How do you undo multiplication? (In other words, what is the opposite of multiplication) \_\_\_\_\_

How do you undo division? (In other words, what is the opposite of division) \_\_\_\_\_

### EXAMPLE 5

Solve:  $2w = 6$ . Solve & identify the property used.



This is asking us  
"Two times what  
number equals 6?"

### EXAMPLE 6

Solve  $\frac{x}{2} = 14$ . Solve & identify the property used.

### EXAMPLE 7

Solve  $-4y = 60$ . Solve & identify the property.

### EXAMPLE 8

Solve  $f \div 3 = 4$ . Solve & identify the property.

**You try it! Solve and identify the property used.**

3)  $x - 8 = -3$

1)  $-\frac{1}{4}x = 5$

4)  $3y = 15$

2)  $6 + x = 5$

# SOLVING TWO STEP EQUATIONS

Review: Solve, check and identify the property.

1.  $-3 + x = -18$

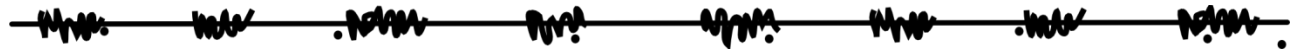
2.  $\frac{p}{3} = 8$

**Two-step equations**- equations that require you to perform two operations in order to

## STEPS TO SOLVE TWO STEP EQUATIONS:

- 1) Draw a line down from \_\_\_\_\_.
- 2) Circle the \_\_\_\_\_.
- 3) Move the term that is \_\_\_\_\_ from the circled variable.  
To do this, **undo** it by performing the \_\_\_\_\_.
- 4) Move (**undo**) the term that is \_\_\_\_\_ the variable by  
performing the \_\_\_\_\_.

ALMOST ALWAYS YOU WILL HAVE TO: 1<sup>ST</sup> UNDO ADDITION/SUBTRACTION  
AND 2<sup>ND</sup> UNDO MULTIPLICATION/DIVISION.



Examples: Solve & identify each property that you use.

1)  $5x + 14 = 74$

3)  $-9 - 4x = 21$

2)  $-3 + \frac{p}{4} = 19$

4)  $\frac{k}{5} - 6 = 3$

REMEMBER: UNDO ADDITION/SUBTRACTION FIRST, AND MULTIPLICATION/DIVISION LAST.

$$5) \frac{s}{6} - 5 = -8$$

$$7) 9 + \frac{k}{5} = 6$$

$$6) 5y + 16 = 51$$

$$8) -12 + 5x = 28$$

REMEMBER: UNDO ADDITION/SUBTRACTION FIRST, AND MULTIPLICATION/DIVISION LAST.

Name: \_\_\_\_\_

# Combining Like Terms

term: \_\_\_\_\_

coefficient: \_\_\_\_\_

**Important:** Whenever a variable does not have a \_\_\_\_\_ it is always an imaginary \_\_\_\_\_!

examples of terms:



like terms:

- terms with the same \_\_\_\_\_ raised to the same \_\_\_\_\_
- \_\_\_\_\_ do not have to be the same

like terms	$3x$ and $2x$	$w$ and $\frac{w}{7}$	5 and 1.4
unlike terms	$3x^2$ and $2x$	$r$ and $\frac{w}{7}$	3.2 and $x$

combining like terms: all you have to do is \_\_\_\_\_ the \_\_\_\_\_!

NOTHING happens to the \_\_\_\_\_. It always STAYS THE \_\_\_\_\_.

non-algebra example: What is 2  + 3  ? Answer: \_\_\_\_\_

algebra example: What is  $2x + 3x$ ? Answer: \_\_\_\_\_

try it!

1) $14b - 8b$	2) $2x^3 + 9x^3$	3) $20x - 2x + 4x$
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# Distributing

**Recall:** The distributive property is all about \_\_\_\_\_.

You **distribute** when a term is directly **in front of** or **behind** a set of \_\_\_\_\_.

The parentheses will have terms inside that are being \_\_\_\_\_ or \_\_\_\_\_.

**examples:**  $6(x + 2)$  and  $(k - 3)7$

**non-examples:**  $6 + (x + 2)$  and  $(k - 3) - 7$

The term on the \_\_\_\_\_ of the parentheses needs to be \_\_\_\_\_ to every term on the inside.

## steps to distribute:

- 1) draw a " \_\_\_\_\_ " from the term on the outside of the ( ) to the first term inside.
- 2) Multiply the two terms that you just connected and write down your answer.
- 3) Draw a " \_\_\_\_\_ " from the term on the outside of the ( ) to the next term inside.
- 4) Multiply the two terms that you just connected and write down your answer.
- 5) Repeat until the term on the outside has been multiplied to each term on the inside.
- 6) Combine like terms if necessary.
- 7) Circle your answer.

Example: 1)  $-8(x + 3) - 15$

2)  $-2(x - 3) + 7x - 9$

try it!

1) $5(9 + w)$	2) $9(7 + p)$	3) $4(5j - 3)$
4) $5(6k + 10) - 15k$	5) $2(3b + 2) - 2b$	6) $3(4c + 2) - 6 - 11c$

# Solving Multi-step Equations

Review:

1)  $4y + 5 = -31$

2)  $-36 + 9n = -27$

A **multi-step** equation is an equation that requires \_\_\_\_\_ in order to solve. Typically you will have to **undo** more than just \_\_\_\_\_. You may encounter problems where you have to \_\_\_\_\_ like \_\_\_\_\_ or \_\_\_\_\_.

## STEPS FOR SOLVING MULTI-STEP EQUATIONS:

\*Note: these steps can be followed for solving one and two step equations also\*

**Step 1:** Draw a \_\_\_\_\_ down from the \_\_\_\_\_.

**Step 2:** \_\_\_\_\_ if needed.

**Step 3:** Combine \_\_\_\_\_.

**Step 4:** \_\_\_\_\_ the variable.

**Step 5:** Undo any \_\_\_\_\_ or \_\_\_\_\_.

**Step 6:** Undo any \_\_\_\_\_ or \_\_\_\_\_.

**Step 7:** \_\_\_\_\_ each step (which means \_\_\_\_\_ the property that you used).

Let's try it!

1)  $2n + 3n + 7 = -41$

2)  $3h - 5h + 11 = 17$

3)  $2(m + 1) - 4 = 16$

4)  $3(t - 12) = 27$

**5)**  $8 + 2k - k = -3$

**6)**  $6a - 2a = -36$

**7)**  $3c - 8c + 7 = -18$

**8)**  $-14 - 7g + 5g = 8$

**9)**  $5(3x + 12) = -15$

**10)**  $\frac{1}{2}(12g - 8) = 26$

**11)**  $2d - 6 + 3d = 14$

**12)**  $5(b + 4) - 6b = -24$

# Equations with Variables on Both Sides

**Review:**

1)  $2(3y + 4) = 20$

2)  $-12 + 6b - 2b = -8$

Sometimes we will encounter problems that have a \_\_\_\_\_ on both sides of the equation.

## Steps for Variables on Both Sides:

**Step 1:** Draw a \_\_\_\_\_ down from the equal sign to separate the two sides.

**Step 2:** \_\_\_\_\_ if needed.

**Step 3:** Combine \_\_\_\_\_ on each side **separately** if needed.

**Step 4:** \_\_\_\_\_ the terms with the variable.

**Step 5:** Move the \_\_\_\_\_ variable to the other side by doing the \_\_\_\_\_.

**Step 5:** Undo any \_\_\_\_\_ or \_\_\_\_\_.

**Step 6:** Undo any \_\_\_\_\_ or \_\_\_\_\_.

**Step 7:** \_\_\_\_\_ each step.



### Let's try it!

1) Solve  $6y + 21 = 9y$

3) Solve  $2(c - 6) = 9c + 2$

2) Solve  $4x + 2x - 24 = 8x$

**4)**  $5a - 12 = 3a + 8$

**6)**  $4b - 13 = 7b - 28$

**5)**  $9x + 4 = 12x - 6x - 11$

**7)**  $2(k + 1) = 3k + 5$

Name: \_\_\_\_\_

Notes #9

## Equations with decimals & fractions

**Review:** Solve the following equations and check:

1)  $2w - 6 = 4w + 8$

2)  $r + 3 = 5r + 19$

When an equation has \_\_\_\_\_ and \_\_\_\_\_, not much changes!

USE YOUR CALCULATOR!!! IT IS YOUR FRIEND!

Convert all fractions to decimals first if you can.

If a fraction is attached to your variable (and it **can't** be converted to a decimal), \_\_\_\_\_

both sides of the equation by the \_\_\_\_\_ to get rid of it. ( \_\_\_\_\_!)

**Let's try it!**

1)  $0.02x + 0.7 = 0.8$

3)  $\frac{2}{3}x + \frac{1}{2} = \frac{5}{6}$

2)  $0.06y + 200 = 0.03y + 350$

4)  $\frac{1}{4}n + 5 = 5\frac{1}{2}$

**Your turn!**

5)  $0.35x + 0.6 = 0.1x + 1$

7)  $\frac{1}{16}x + \frac{1}{4} = \frac{1}{2}$

6)  $2(x - 3) = 1.2 - x$

8)  $3.3 - x = 3(x - 1.7)$

# solving one-step inequalities

## (SHOULD BE) REVIEW

$>$	$<$	$\leq$	$\geq$

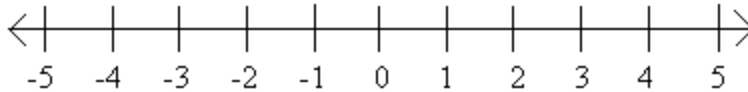
When graphing, if \_\_\_\_ or \_\_\_\_, the circle will be \_\_\_\_\_ on the value.

When graphing, if \_\_\_\_ or \_\_\_\_, the circle will be \_\_\_\_\_ on the value.

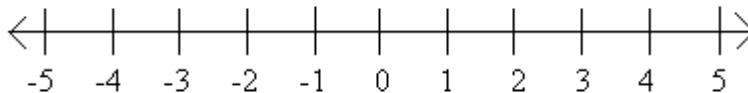
Depending on the inequality sign: \_\_\_\_\_ in the direction of the values that satisfy the inequality.

### Examples:

1)  $a < 2$



2)  $3 \geq d$



Just like with equations, our goal is to \_\_\_\_\_ the variable.

### Examples:

Let's solve, number line and explain each of the following inequalities.

1)  $x + 3 > 10$

2)  $-12 \geq 7 + x$



3)  $6 < 2k$

4)  $\frac{x}{2} \geq 4$

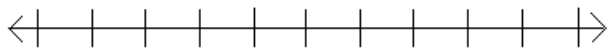




Solve each inequality and graph it on a number line. Explain the solution.

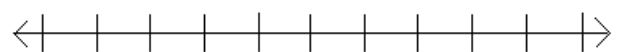
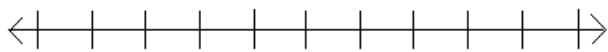
1)  $b + 7 \leq -4$

2)  $-5 < n - 3$



3)  $3x > -18$

4)  $-2 \leq \frac{x}{4}$



5)  $-12 + c < 14$

6)  $\frac{x}{3} \geq 5$



# Solving Two-Step Inequalities

Consider the inequality  $4 > 1$ .

1. Complete each statement on the right by choosing  $<$  or  $>$ .
2. What happens to the inequality symbol when you multiply each side by a positive number?
3. What happens to the inequality sign when you multiply each side by a negative number?

$4 \cdot 3$ _____ $1 \cdot 3$
$4 \cdot 2$ _____ $1 \cdot 2$
$4 \cdot 1$ _____ $1 \cdot 1$
$4 \cdot -1$ _____ $1 \cdot -1$
$4 \cdot -2$ _____ $1 \cdot -2$
$4 \cdot -3$ _____ $1 \cdot -3$

**Solve and Compare:**

1.)  $3x > 9$

2.)  $-3x > 9$

If you multiply or divide each side of an inequality by a \_\_\_\_\_ number, the inequality stays in the same direction.

If you multiply or divide each side of an inequality by a \_\_\_\_\_ number, the inequality **flips directions**.

**Solve, graph, and explain the solution**

1)  $2k + 4 < 6$

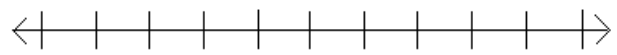
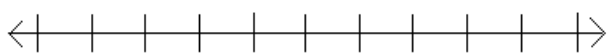


2)  $-3m - 6 < 9$



3)  $4 \leq \frac{x}{2} - 3$

4)  $-\frac{x}{3} + 2 \geq 1$



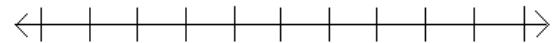
Name: \_\_\_\_\_

**Homework #11**

**EXAMPLES:** Solve and graph and explain the solution.

1)  $3g + 9 < 18$

2)  $\frac{k}{-2} + 5 \leq -4$



What numbers are a part of the solution set? Circle all that apply. (there can be more than one)

3.)  $-5c + 9 < -11$

a) 6

b) -3

c) -4

d) 4

4.)  $\frac{x}{3} - 1 \leq 2$

a) 3

b) 9

c) 4

d) 10

Name: \_\_\_\_\_

Homework #10

Solve the following inequalities. **Graph and explain the solution.**

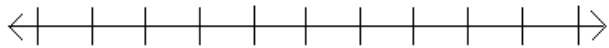
1.)  $n - 7 > 2$

2.)  $x + 1 < -3$



3.)  $60 < 12b$

4.)  $-5 \geq \frac{t}{3}$



5.)  $6 < y - 3$

6.)  $16d > -64$



7.)  $\frac{w}{7} > 0$



Name \_\_\_\_\_

Homework #9

**Solve each problem and JUSTIFY each step.**

$$\#1) 2(x+2) = \frac{9}{2}$$

$$\#2) 0.3n + 4.1 = -0.6n - 1.2$$

$$\#3) \frac{1}{3} \left( \frac{3}{5}w + \frac{12}{10} \right) = \frac{4}{5}w + \frac{1}{10}$$

$$\#4) 0.4(m+0.7) = 1.5$$

Name: \_\_\_\_\_

Homework #8

Solve the equation and justify each step.

1.  $5 + 4x = x + 8$

2.  $3z + 7 = 2(z + 5)$

3.  $-2(6 - 2m) = 3m - 8 + 5m$

4.  $3(x - 2) - 2x = 4x + 9$

Name: \_\_\_\_\_

Homework #7

Solve the following equations and identify the properties used.

1)  $3(d + 2) = 6$

2)  $26 = 2(m + 10)$

3.)  $18 = 2m + 6 - 5m$

4.)  $2(b - 3) - 4b = 4$



Use the Distributive Property  
& Combine Like Terms

Name: \_\_\_\_\_

1.)  $3 - 3(x - 2)$

2.)  $-(1 - 5n) - 7n$

3.)  $8 + 7(7n - 4)$

4.)  $4x + 5(3x - 3)$

5.)  $5 - 2(8x + 4)$

6.)  $1 - 8x - 5x$

7.)  $7 + 6x + 9x + 9$

8.)  $-3 + 8x + 2$

9.)  $5 - 8n - 4n$

10.)  $9 + 3x + 1 - 2x$

Name: \_\_\_\_\_

Homework #5

Solve the following 2 step equations and CHECK! Then identify the properties used.

1.)  $2c + 5 = 35$

2.)  $\frac{p}{4} + 3 = 15$

3.)  $34 = 14n - 8$

4.)  $5y + 16 = 51$

4.)  $-\frac{m}{9} + 7 = 3$

6.)  $\frac{n}{4} - 3 = 6$

7.)  $-3 = -3k + 6$

8.)  $-9 = -\frac{s}{12} + 5$

Name: \_\_\_\_\_

Homework #4

Solving one-step equations

You must **show your work** to get credit. Identify the property you used.

1)  $y + 9 = 23$

2)  $\frac{x}{4} = 16$

3)  $-78 + z = 100$

4)  $-8c = 96$

5)  $3n = 39$

6)  $10 = a - 15$

7.)  $48 + b = 56$

8.)  $\frac{1}{5}x = 4$

Name: \_\_\_\_\_

Homework #3

1.) Determine the set or sets the following numbers belong to:

-3.2      -35                  7                  0                   $\sqrt{8}$                    $\frac{3}{5}$                   .23974...

Natural:

Whole:

Integer:

Rational:

Irrational:

2.) Which statement is *not* always true?

- (1) The product of two irrational numbers is irrational.
- (2) The product of two rational numbers is rational.
- (3) The sum of two rational numbers is rational.
- (4) The sum of a rational number and an irrational number is irrational.

3.) Given:  $L = \sqrt{2}$   
 $M = 3\sqrt{3}$   
 $N = \sqrt{16}$   
 $P = \sqrt{9}$

Which expression results in a rational number?

- |             |             |
|-------------|-------------|
| (1) $L + M$ | (3) $N + P$ |
| (2) $M + N$ | (4) $P + L$ |

Name: \_\_\_\_\_

Homework #2

Identify the property that each example illustrates.

- 1 Which property is illustrated by the equation  $ax + ay = a(x + y)$ ?
- 2 Which property of real numbers is illustrated by the equation  $-\sqrt{3} + \sqrt{3} = 0$ ?
- 3 Which property of real numbers is illustrated by the equation  $52 + (27 + 36) = (52 + 27) + 36$ ?
- 4 If  $M$  and  $A$  represent integers,  $M + A = A + M$  is an example of which property?
- 5 Which property is illustrated by the equation  $\frac{3}{2}x + 0 = \frac{3}{2}x$ ?
- 6 Which property is illustrated by the equation  $6 + (4 + x) = 6 + (x + 4)$ ?
- 7 What is the multiplicative inverse of  $\frac{3}{4}$ ?
8. Under which operation is the set of odd integers closed?
  - 1) addition
  - 2) subtraction
  - 3) multiplication
  - 4) division