Name:_____ Quiz 8: Logs in Modeling Review Sheet

1) Solve the following exponential equations.

a. $10^{x^2+5} = 100^{2x^2+x+2}$

$10^{x^2+5} = 10^{2(2x^2+x+2)}$

$x^2+5 = 4x^2+2x+4$

$-x^2 \quad -5 \quad -x^2 \quad -5$

$0 = 3x^2 + 2x - 1$

way 1: $0 = (3x-1)(x+1)$

$3x-1=0 \quad +1 \quad +1$ | $x+1=0 \quad -1 \quad -1$

$\frac{3x}{3} = \frac{1}{3}$ | $x = -1$

$x = 1/3$

way 2:

$x = \frac{-2 \pm \sqrt{(2)^2-4(3)(-1)}}{2(3)}$

$x = \frac{-2 \pm \sqrt{16}}{6}$

$x = \frac{-2+4}{6}$ → $x = \frac{-2+4}{6} = \frac{2}{6} = 1/3$ ✓

$x = \frac{-2-4}{6}$ → $x = \frac{-2-4}{6} = \frac{-6}{6} = -1$ ✓

b. $3^{\frac{x}{5}} = 2^{4x-2}$

$\log(3^{x/5}) = \log(2^{4x-2})$

$\frac{x}{5}\log(3) = (4x-2)\log(2)$

$\frac{x}{5}\log(3) = 4x\log(2) - 2\log(2)$

$-4x\log(2) \quad -4x\log(2)$

$\frac{x}{5}\log(3) - 4x\log(2) = -2\log(2)$

$x\left(\frac{1}{5}\log(3) - 4\log(2)\right) = -2\log(2)$

$\frac{x\left(\frac{1}{5}\log(3) - 4\log(2)\right)}{\left(\frac{1}{5}\log(3) - 4\log(2)\right)} = \frac{-2\log(2)}{\left(\frac{1}{5}\log(3) - 4\log(2)\right)}$

$x \approx .543$

you can simplify the logs

$x = \frac{-2\log(2)}{\frac{1}{5}\log(3) - 4\log(2)}$

Not necessary, but optional:

$x = \frac{\log(2^{-2})}{\log(3^{1/5}) + \log(2^{-4})}$

$x = \frac{\log\left(\frac{1}{4}\right)}{\log(\sqrt[5]{3}) + \log\left(\frac{1}{10}\right)}$

2) Solve the following system of equations.

$4^{-3x+y} = 16$

$6^{x+y} = 46656$

$4^{-3x+y} = 4^2$

$6^{x+y} = 6^6$

$-3x+y=2$

$+3x \quad +3x$

$y = 2+3x$

$x+y=6$

$x+2+3x=6$

$-2 \quad -2$

$\frac{4x}{4} = \frac{4}{4}$

$x=1$

$y=2+3(1)$

$y=5$

$(1,5)$

3) Use properties of logarithms to solve the equation $f(x) = g(x)$. Give your answer as a logarithmic expression, and approximate it to two decimal places.

$f(x) = 100^{x^3+x^2-4x}, g(x) = 10^{2x^2-6x}$

way 1:

$\log(100^{x^3+x^2-4x}) = \log(10^{2x^2-6x})$

$(x^3+x^2-4x)\log(100) = (2x^2-6x)\log(10)$

$(x^3+x^2-4x)2 = 2x^2-6x$

$2x^3+2x^2-8x = 2x^2-6x$

$+6x \quad +6x$

$2x^3-2x=0$

$2x(x^2-1)=0$

$2x(x+1)(x-1)=0$

$2x=0 \quad x+1=0 \quad x-1=0$

$x=0 \quad x=-1 \quad x=1$

way 2:

$100^{x^3+x^2-4x} = 10^{2x^2-6x}$

$10^{2(x^3+x^2-4x)} = 10^{2x^2-6x}$

$2x^3+2x^2-8x = 2x^2-6x$

$+6x \quad +6x$

$2x^3-2x=0$

$2x(x^2-1)=0$

$2x(x+1)(x-1)=0$

$2x=0 \quad x+1=0 \quad x-1=0$

$x=0 \quad x=-1 \quad x=1$

$x=-1,0,1$ or $x=0,\pm1$