Name: ________________________________

MODULE 3 TEST REVIEW SHEET

1) Use properties of exponents to explain why it makes sense to define $8^{\frac{1}{3}}$ as $\sqrt[3]{8}$.

$\frac{\text{power}}{\text{root}} = \sqrt[3]{8^1}$

2) Use properties of exponents to rewrite each expression as either an integer or as a quotient of integers $\frac{p}{q}$ to show the expression is a rational number.

a. $16^{\frac{3}{2}} \cdot \left(\frac{1}{27}\right)^{\frac{2}{3}}$

$\sqrt[2]{16}^{\,3} \cdot \sqrt[3]{\left(\frac{1}{27}\right)}^{\,2}$

$(4)^3 \cdot \left(\frac{1}{3}\right)^2$

$64 \cdot \frac{1}{9} = \boxed{\frac{64}{9}}$

b. $\frac{\sqrt[3]{16}}{\sqrt[3]{2}}$

$\sqrt[3]{\frac{16}{2}} = \sqrt[3]{8} = \boxed{2}$

3) Use properties of exponents to find the numerical value of each expression when $x = 3, y = 2$, and $z = 4$.

a. $\sqrt{\frac{xy^2}{(x^3z)^{\frac{1}{2}}}}$

$= \sqrt{\frac{(3)(2)^2}{((3)^3(4))^{1/2}}}$

$= \sqrt{\frac{12}{(108)^{1/2}}}$

$\boxed{\approx 1.075}$

b. $\sqrt[11]{y^2z^4}$

$\sqrt[11]{(2)^2(4)^4}$

$\sqrt[11]{(4)(256)}$

$\boxed{\approx 1.878}$

c. $x^{-\frac{3}{2}}y^{\frac{4}{3}}z^{-\frac{3}{4}}$

$= (3)^{-3/2}(2)^{4/3}(4)^{-3/4}$

$\boxed{\approx .1715}$

4) Solve each equation. Express your answer as a logarithm, and then approximate the solution to the nearest thousandth.

a. $\log(2)^{3x} = \log 9$

$$\frac{\cancel{3}x \log(2)}{\cancel{3}\log(2)} = \frac{\log(9)}{3\log(2)} \qquad x = \frac{\log(9)}{3\log(2)}$$

$\boxed{x = 1.057}$

b. $\frac{\cancel{30}e^{0.3t}}{\cancel{30}} = \frac{90}{30}$

$\ln(e^{.3t}) = \ln 3$

$$\frac{\cancel{.3}t \cancel{\ln(e)}}{\cancel{.3}} = \frac{\ln(3)}{.3} \qquad t = \frac{\ln(3)}{.3}$$

$\boxed{t = 3.662}$

c. $\frac{\cancel{15}\left(10^{\frac{t}{12}}\right)}{\cancel{15}} = \frac{52}{15}$

$\log(10^{t/12}) = \log\left(\frac{52}{15}\right)$

$\cancel{(12)}\frac{t}{\cancel{12}}\log(10) = \log\left(\frac{52}{15}\right)(12)$

$t = 12\log\left(\frac{52}{15}\right)$

$\boxed{t \approx 6.479}$

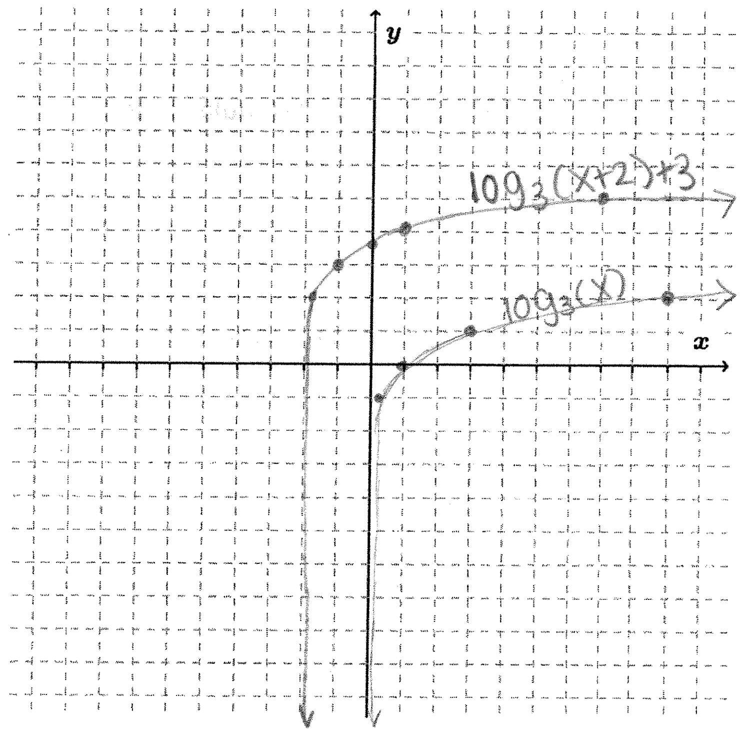
5) Sketch the graph of each pair of functions on the same coordinate axes and describe the graph of g as a series of transformations of the graph of f.

a. $f(x) = \log_3(x)$, and $g(x) = \log_3(x+2) + 3$

left 2 up 3

X	y
0	error
$\frac{1}{3}$	-1
1	0
3	1
9	2
~~27~~	~~3~~

X	Y
0	3.63
$-\frac{5}{3}$	2
-1	3
1	4
7	5
~~25~~	~~8~~



6) Find the inverse g for each function f.

b. $f(x) = \frac{1}{5}x + 9$

$x = \frac{1}{5}y + \cancel{9}$

$-9 \quad -9$

$(5)\ x - 9 = \frac{1}{5}y\,(5)$

$\boxed{y = 5x - 45}$

c. $f(x) = 3^{2x} - 1$

$x = 3^{2y} \cancel{-1}$

$+1 \quad +1$

$\log(x+1) = \log(3^{2y})$

$\frac{\log(x+1)}{2\log(3)} = \frac{2y\log(3)}{2\log(3)}$

$y = \frac{\log(x+1)}{(2\log(3))}$

$1 + \frac{.05}{12} = 1.0042$

7) Stephanie has $1,500 in an investment account that earns 5% per year, compounded monthly.

a. Write a recursive sequence for the amount of money in her account after n months.

$a_0 = 1500 \quad a_n = a_{n-1}(1.0042)$

← real world example so we use a_0

b. Write an explicit formula for the amount of money in the account after n months.

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = 1500\left(1 + \frac{.05}{12}\right)^{12t}$

$A = 1500(1 + .0042)^{12t}$

$A = 1500(1.0042)^{12t}$

$a_n = 1500(1.0042)^n$ ← real life example, so exponent is n, not $n-1$

month of $\frac{1}{12}$ of year

c. Write an explicit formula for the amount of money in her account after t years.

$\rightarrow a_n = 1500(1.0042)^{12t}$

same thing

in years

d. How much money will she have in her account in 15 years?

$a_{15} = 1500(1.0042)^{12 \cdot 15}$

$\boxed{a_{15} = \$3189.56}$

8) Consider the functions $f(x) = 3^{2x+3}$ and $g(x) = 5^{3x}$.

a. Use properties of logarithms to solve the equation $f(x) = g(x)$. Give your answer as a logarithmic expression, and approximate it to two decimal places.

$\log(3^{2x+3}) = \log(5^{3x})$

$(2x+3)\log(3) = 3x\log(5)$

$2x\log(3) + 3\log(3) = 3x\log(5)$

$-2x\log(3) \qquad -2x\log(3)$

$3\log(3) = 3x\log(5) - 2x\log(3)$

$\frac{3\log(3)}{3\log(5) - 2\log(3)} = \frac{x(3\log(5) - 2\log(3))}{3\log(5) - 2\log(3)}$

$x = \frac{3\log(3)}{3\log(5) - 2\log(3)} \approx \boxed{1.25}$